Computer-Supported Collaborative Learning
DOI 10.1007/s11365-006-9658-y

Student participation in math discourse with networked handhelds

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Received: 3 November 2005 / Revised: 29 June 2006 /
Accepted: 30 June 2006 / Published online: 00 Month 0000

Abstract This study explores the potential of networked handheld computers to support collaborative problem solving in small groups. Drawing on data from a middle school mathematics classroom equipped with a wireless handheld network, I argue that the sharing of mathematical objects through interactive devices broadens the "bandwidth" of classroom collaboration, expanding the range of participatory forms through which students might contribute to the work of a group and enhance their own learning. The analysis focuses on the participation strategies of those students in two focus groups who were most able to demonstrate posttest score gains from relatively low scores on a pretest. In particular, the device network provided those students with a set of collective, dynamic objects through which they supplemented and coordinated discursive forms of participation in the joint work of their respective groups.

Keywords Classroom networks · Mathematics · Mobile devices · Small group pedagogy

Introduction

Pointing to such features as wireless connectivity, easy portability and relatively low cost, some observers have argued that handheld computers offer the potential to support student learning much more broadly than desktops (Roschelle & Pea, 2002; Soloway et al., 2001). Certainly, mobile devices ought to lend themselves to more frequent and more varied uses; classroom handheld networks present students with computing tools that are at once highly personal and highly interactive, facilitating a

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wide array of activity structures and instructional modes. But which among those implementations of networked devices might constitute effective pedagogies? And to what extent would they afford meaningful learning opportunities, and equitable access to those opportunities?

To address those questions, this paper explores the potential of a handheld network to support collaborative problem solving in small groups. Drawing on data from a study of middle school mathematics classes, I argue that the sharing of mathematical objects through networked devices broadens the "bandwidth" of classroom collaboration, expanding the range of participatory forms through which students might contribute to the work of a group. In particular, transactions involving dynamically linked artifacts distributed across a set of devices can supplement spoken exchanges with an additional layer of networked interactions, including opportunities to observe, contribute to and coordinate problem-solving processes and discourse. The analysis below uses detailed examinations of students' utterances and network transactions during problem-solving activities as resources for interpreting performance on an independent achievement measure, suggesting important links between network-supported participation and opportunities for mathematical learning.

Supporting collaboration through classroom networks

Early investigations of interactive handheld use outlined a number of promising designs for classroom collaboration (i.e., DiGiano et al., 2003, Pinkwart, Hoppe, Milrad, & Perez, 2003). Kaput (2000) notes that such instructional designs for networked devices inherit long pedagogical traditions of classroom activity, organized at one of three levels—the whole (teacher-led) class, small groups, or individual students. Classroom networks overlay these social organizations with corresponding distributions of shared objects across devices. To date, most research efforts involving designs for interactive devices have emphasized either whole-class level aggregation of student contributions to jointly create shared objects in a collective network space (Wilensky & Stroup, 1999), or question-response transactions between the teacher and each individual student (Davis, 2003; Roschelle, 2003). A few researchers have begun to explore ways in which networked devices might support the sharing of objects at the level of small collaborative groups. One such design, set in a non-networked context, involves activities in which pairs of students exchange functions between devices, through either linking cables or infrared beaming, in order to discursively negotiate the matching of graphs for those functions (Roschelle, Vahey, Tatar, Kaput, & Hegedus, 2003). Hegedus and Kaput (2004) describe another scenario in which members of a group use their devices to contribute different parameter values in order to generate families of functions on a shared display.

Though explorations of designs for small groups with these emerging technologies are still in very preliminary stages, the distinctive affordances of a classroom network may be particularly well suited to small group pedagogy. Interactive devices make possible the layering of students' discursive communication with simultaneous transactions through the network, potentially allowing students to contribute to collective processes through channels that were unavailable in conventional classrooms (Ares & Stroup, 2004). In whole-class activities and discussion, however, device-level contributions may constitute the only or the primary form of participation in those collective processes; sheer numbers in a classroom of 25 or more students minimize the opportunities for any single student's discursive participation in a teacher-directed conversation centered on a shared display. Small groups of two to four students, on the other hand, should provide opportunities for each student to couple network-level transactions of shared objects with extensive discursive participation. In such contexts, the small screen size of handheld computers, often cited as a limitation of such devices, may in fact prove to a be a resource for facilitating the near-simultaneous management of face-to-face and device-level engagement.

Roschelle and Teasley (1995) characterize collaboration as the construction and maintenance of a "joint problem space", and (desktop) computer-based learning environments as resources for mediating that collaboration. In such an environment, a pair of students collaborating in front of a single machine shares a view of objects around which, and a representational context within which, to establish and negotiate collective meanings and convergent interpretations for the phenomena that structure a joint problem space. In a networked environment, by contrast, the relevant objects and representations of a problem context are shared across the respective devices of two or more students, potentially restructuring the mediating role those devices might play. While objects on a common desktop screen are shared in the trivial sense of an identical view provided for multiple observers, objects distributed across multiple devices may appear in different views, configurations or representational settings. Sharing under the latter circumstances moves from a trivial, identical relationship to a potentially ambiguous and complex one, in which coordinating collective meanings may require considerably more discursive negotiation.

Simply conducting those negotiations may present ample learning opportunities. Coordinating conversations around shared objects in different representational settings, for example, may highlight the invariant properties that characterize those objects across those settings (White, 2005). Moreover, distributed configurations in a networked small group can equip students to orient in joint problem spaces that resemble complex, authentic tasks from beyond the classroom. Hutchins' (1995) exquisitely detailed account of the joint computational efforts of groups of people and tools navigating large ships, for example, provides one compelling illustration of a collaborative task characterized by the propagation of representational states across a distributed array of multiple media and participants. Challenging students to undertake similar tasks with networked devices may constitute a particularly powerful opportunity to disrupt conventional instructional emphases on unaided and individual performances by "teaching for the design of distributed intelligence" (Pea, 1993, p. 81).

Divisions of labor: equitable access to collaborative learning opportunities

Importantly, distributions are not always equitable; the shared and collective nature of phenomena and meanings in a collaborative environment does not ensure that all students will contribute equally to joint tasks. In a comparative examination of the computer-supported collaborative practices of adults in workplace tasks, and of students engaged in school-based mathematics problem solving activities, Stevens (2000) found that remarkably similar divisions of labor emerged in each setting. In the classroom case, those divisions led to some students' developing of competencies and understandings that were more highly prized in the formal assessment practices of the classroom than those developed by other students in the group. Of course, the workload to be divided in a shared task is a multifaceted phenomenon. Barron (2003) has pointed to the utility of conceptualizing collaborative student work as a "dual-problem space" in which students must simultaneously orient toward "a *content space* (consisting of the problem to be solved) and a *relational space* (consisting of the interactional challenges and opportunities)" (p. 310, emphasis in original). From this perspective, emergent divisions of labor in a collaborative classroom task should reflect students' interactions, both with the relevant content and with one another. The extent to which each student in a group might participate in and learn from collaborative tasks thus depends on their success in simultaneously navigating content and relational dimensions. Social status, for example, may prove as pivotal as domain competence in structuring an individual student's opportunities to contribute to and learn from a collaborative group. One crucial question for group designs with interactive devices involves whether status might be reorganized in the networked space, potentially affording new resources and opportunities for student collaboration and learning.

Cohen (1986, 1994) provides one set of pedagogical tools for organizing student small groups into effective and equitable learning spaces. Key elements in her approach include the assignment of roles that specify distinctive ways for each member to contribute to the group, and tasks that necessitate the combined efforts and resources of students in each of those roles. The balancing of role contributions and task demands in the design and implementation of collaborative problem-solving activities represents a way to structure divisions of group labor so as to treat rather than reproduce problems of status. In a classroom handheld network, the distribution of those various roles and resources across students' respective devices might both facilitate the need for collective action and provide opportunities for students to participate through the network as well as discursively. In the remainder of this paper, I describe the design and present results from the initial implementation of a system intended to capitalize on these affordances of interactive devices in order to support collaborative mathematics learning.

The *code breaker* learning environment

This paper draws on data collected during the first classroom implementation of a learning environment situated in a classroom network of wireless handheld computers. Intended to support mathematical learning through collaborative problem-solving activities, this designed environment attempted to capitalize on two features of the handheld devices: their capacities to simultaneously display multiple linked representations of a mathematical function, and to connect multiple students through a local wireless device network. A handheld client application, called *Code Breaker*, allowed each student to edit parameters of a polynomial function and to examine corresponding changes in an array of graphical, tabular, numerical and other linked displays, each of which provided different resources relevant to an open-ended problem-solving task. Through a desktop server application (see Fig. 1), the teacher assigned the devices of students who were seated together in groups of four to a corresponding server-defined group so that changes to the function on one student's device automatically propagated to the devices of the other group members.

A curricular unit accompanying this handheld network asked students to imagine themselves as cryptographers and to collaborate with the other members of their small group on daily problem-solving activities involving the making and breaking of codes. To generate these codes, letters in the alphabet were assigned to their ordinal values 1 through 26, and then mapped through a polynomial encoding function to produce a set of output values comprising a numerical cipher text alphabet. Decryption activities commenced when a group downloaded a string of numbers representing a message that had been encrypted by the teacher or by another student group. The problem-solving process involved using the *Code Breaker* software to match an editable "candidate" function to the unknown encoding function from which the encrypted message had been generated. The candidate and encoding functions thus constituted shared mathematical objects linking the various representational states of each student's device. Contributing to the problem-solving process of the group required reporting, interpreting, or altering the state of one of those objects.

Each student in a group was assigned a role that included responsibility for viewing certain representational artifacts, only a few of which could be simultaneously displayed on a single student's device[1] (Fig. 2). These roles were rotated daily and were designed to promote collaboration by providing multiple entry points into the

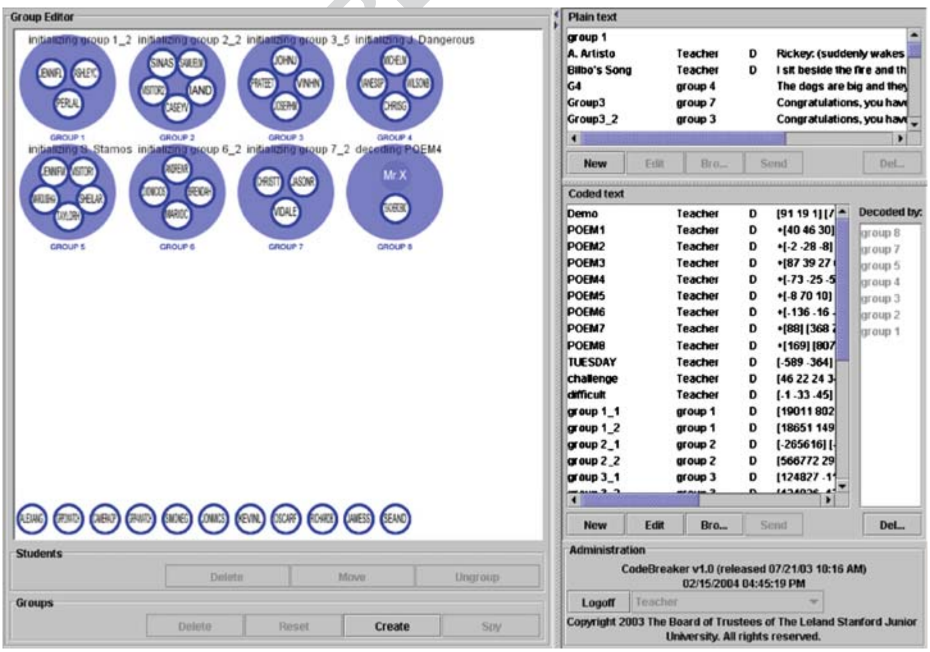

Fig. 1 *Code breaker* teacher's view

problem-solving process, as well as to emphasize connections among various function representations. The student assigned to the role of "Publisher" was responsible for editing the candidate function, and also had views of a graph of that function and of the cipher text message. The group's "Presenter" shared the Publisher's views of the

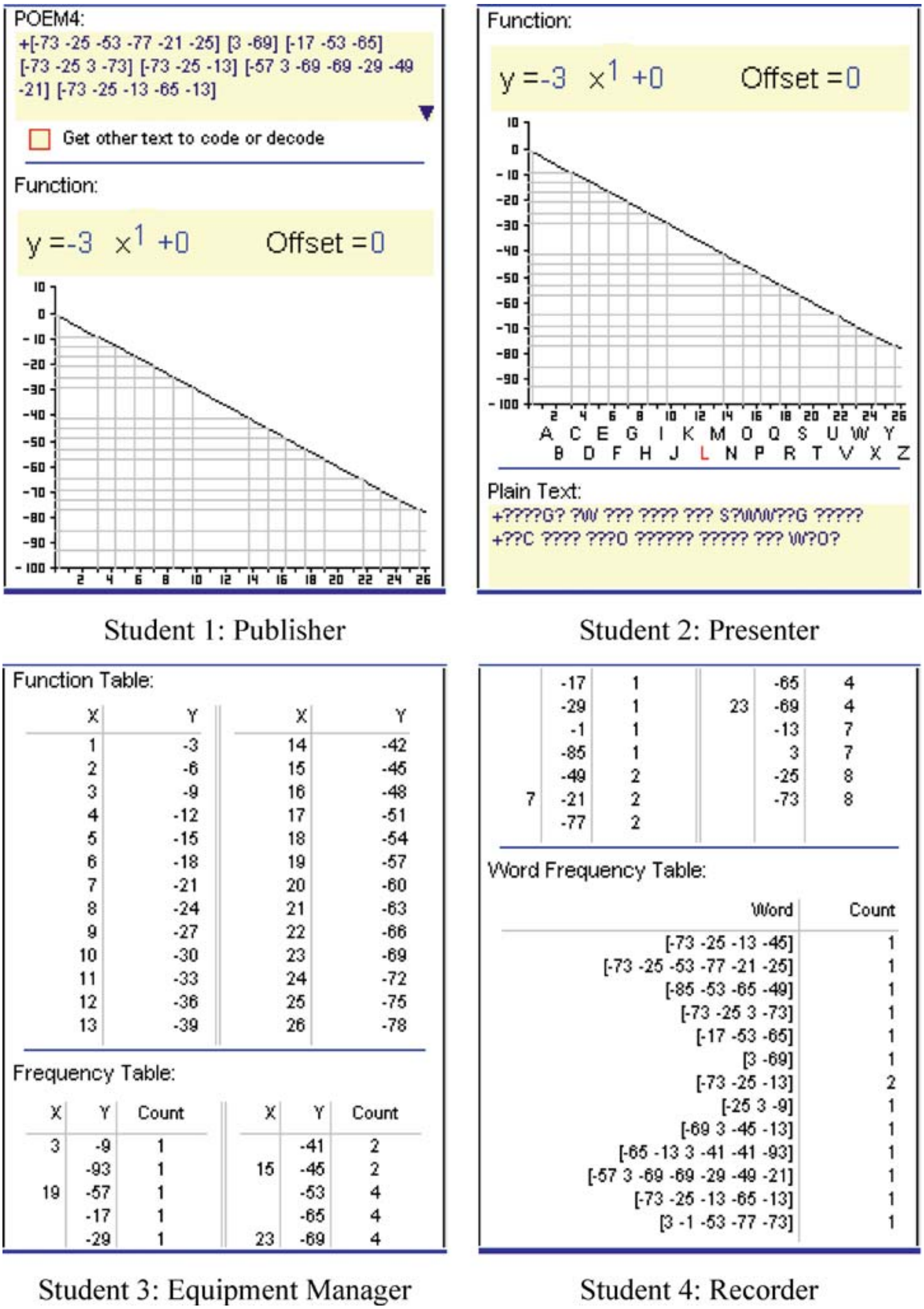

Fig. 2 *Code breaker* student roles and views

candidate function and its graph, as well as an image of the plaintext translation to which the inverse of the current candidate function would map the cipher text. The "Equipment Manager" took responsibility for examining a tabular representation of the candidate function, while the "Recorder" analyzed tables displaying frequencies of characters and words in the cipher text message. As their names suggest, the Presenter, Recorder and Equipment Manager roles also corresponded to other classroom responsibilities, such as recording the group's problem-solving progress in a notebook, or presenting successful solution strategies to the whole class. Only the responsibilities of the Publisher were focused exclusively on the decryption tasks, because that student played a particularly central role in the group's problem-solving efforts by controlling the state of the candidate function. Consequently, that role was of particular interest in relation to affording opportunities for student participation, and will be examined in detail below.

Instructional context

The *Code Breaker* handheld environment was implemented with two middle school mathematics classes during a five-week summer school session. The student population was highly diverse in terms of prior academic achievement. While some students were required to attend the summer program due to poor math performance during the prior school year, others were high-attaining students voluntarily participating in the course for enrichment. Collaborative groups were organized to reflect that academic diversity; student assignments to their groups were based partly on a pretest score, so that each group included students who ranged widely in their performance levels on that instrument, and partly on the observations of the teacher and researchers. Whenever possible, groups remained the same for the duration of the study. The first two weeks of the unit were devoted to introducing the history, terminology and principles of cryptography, as well as the mathematics of simple polynomial functions and the mechanics of the handheld computers and the *Code Breaker* client software. Thus equipped, students spent much of their class time over the remaining three weeks of the unit working on code breaking activities in their small groups.

Research methods

This paper uses in-depth case studies of two student groups to investigate the forms of access to collaborative mathematics problem solving afforded by the *Code Breaker* environment. The primary method of this study involved interactional analysis of two collaborative groups: Group A, comprised of four boys (CJ, Jason, Reggie and Vince), and Group B, comprised of four girls (Jessica, Monique, Shirley, and Tina). These groups were purposively selected according to several criteria, including the consent of all members to be videotaped, and informal observations of their levels of on-task discourse during preliminary activities.[2] The following analysis draws from video data of student interactions in a total of 32 decoding "events" undertaken by the two groups over the final three weeks of the study. These events, ranging from 2 to 30 min in duration, began when a group downloaded a new code to break, and concluded when they either solved or stopped working on the code. All decoding events were transcribed, and each student utterance during these events was coded to identify the way the speaker was discursively interacting—or not—with her group mates. These codes included “instructions” issued from one student to another, and “responses” indicating follow-up on an instruction; “explanations” and “observations”, as well as “replies” to either of these; “questions” and their corresponding “answers”. These codes accounted for 2,416, or nearly 80%, of 3,030 total utterances made by these eight students over 32 decoding events; the remaining 614 utterances were either off-topic or non-interactive remarks that did not appear to be either directed at or acknowledged by other students.

This qualitative analysis is complemented by a report of students’ performances on pre- and posttests administered during the study. In order to provide a gauge for learning of relevant mathematical skills and concepts, students were asked to complete paper-based assessments of their understanding of mathematics related to the *Code Breaker* unit at the beginning and at the end of the five-week session. Topics included reading graphs and tables, evaluating arithmetic and simple algebraic expressions, solving linear equations, and vocabulary words related to functions, algebraic expressions and graphs. The test included 45 total items—27 multiple choice problems, nine vocabulary questions, and nine items involving filling in or interpreting function tables. Several of the multiple choice questions were drawn or adapted from release items from a statewide, standardized test, while other questions were developed specifically to assess students’ learning of content emphasized in the *Code Breaker* unit. These assessments were not intended to evaluate the effectiveness of the learning environment or the instructional unit, especially given the absence of a control. Rather, patterns in student performance served to illustrate trends in relation to student participation in collaborative problem solving highlighted by the interactional analysis.

Learning in networked groups: forms of participation and performance

The analysis that follows blends three distinct strands, taken up in turn below, to compose a framework for interpreting the different ways students managed to participate in collaborative groups to solve problems in the *Code Breaker* environment. I begin by summarizing pre- and posttest data for the eight focus students; the remainder of the paper is devoted to unpacking those performances. The next section examines students’ discursive forms of participation in the group, comparing the assessment results with patterns in students’ utterances during decoding activities. I relate one form of participation, the issuing of utterances coded as instructions, to score increases from pre- to posttest, and consider the extent to which an assigned student collaborative role scaffolded that form of participation. The final section draws on detailed analyses of student interactions with both their devices and with one another in three brief episodes in order to illustrate the potential significance of students’ participation through non-discursive and network-supported forms.

Reflecting efforts to organize groups heterogeneously by achievement, the students in the two focus groups displayed considerable score variation on both the pre- and posttests. As Table 1 indicates, the extent to which students improved their performances from pre- to posttest varied considerably as well. These results will form the lens through which I subsequently examine the various ways students

Table 1 Pre- and posttest comparisons (# of correct answers out of 45 total items)

	Class mean	Pair 1		Pair 2		Pair 3		Pair 4	
		Tina	CJ	Vince	Jessica	Jason	Reggie	Shirley	Monique
Pretest	22.6	42	37	29	30	21	21	12	14
Posttest	26.2	45	44	37	34	30	23	25	18
Raw gain	+3.6	+3	+7	+8	+4	+9	+2	+13	+4
Gain score	0.16	1.00	0.88	0.50	0.27	0.38	0.08	0.39	0.13

participated in their groups' problem-solving efforts. Table 1 pairs students from the two focus groups according to their pretest performance. By that measure, the four pairs are closely matched. Tina and CJ had the two highest pretest scores of the focus students and among the highest in their respective classes; Vince and Jessica's nearly identical pretest scores were near the 75th percentile for the two classes; Jason and Reggie's matching scores were at the class median and just below the mean; and Shirley and Monique both performed below the lower quartile.

Many of these pretest similarities within pairs faded on the posttests. While Tina and CJ both did well on the pretest and even better on the posttest (missing only one point between them), the performances of the students in the other three pairs diverged over the course of the unit. In each case, one of the two students managed only a modest score increase on the posttest, while the other posted a considerable gain. While Jessica and Monique improved their raw scores by four points each and Reggie by two points, all close to the mean class gain of 3.6 points, Vince, Jason, and Shirley were all well above that mean increase. Vince and Jason raised their raw scores by eight and nine points, respectively, and Shirley's score increase of 13 points was the highest among any of the 44 students tested in two classes. Variations in gain scores within pairs highlight these differences even more clearly, with Vince making up 50% of the gap between his pretest score and a perfect 45 points as compared to 27% of the corresponding gap for Jessica, Jason 38% to Reggie's 8%, and Shirley 39% to Monique's 13%.

To be sure, these score increases provide only a narrow window into the students' learning over the course of the unit. But as the analyses below suggest, these divergent results closely correspond to trends in other measures of student performance over the course of the unit. Together, these quantitative and qualitative reflections of varying student experiences during the period of students' work in the *Code Breaker* environment reveal forms of student participation in collaborative tasks that appeared to be particularly supportive of their mathematical learning.

Divisions of code-breaking labor: patterns in discourse

The posttest score gains achieved by the eight focus students closely reflected patterns in student discourse highlighted in the coding scheme. In particular, those students with greatest achievement gains had more often directed the flow of problem-solving activity, whereas those who made only modest increases more often followed those directions. Table 2 highlights this trend, displaying the percentage of each student's interactive utterances that were coded as instructions and responses, respectively. Clearly, the respective frequencies with which the students gave and responded to instructions varied considerably. Three students, CJ, Tina and Vince, issued instructions much more frequently than their peers, at rates of 35, 45 and 37%, respectively. These three students also had the three highest posttest scores of the focus cohort, and also the three highest gain scores. In addition to giving instructions more frequently than their peers, they rarely responded to the instructions of others, doing so a considerably smaller percentage of the time than the other five students. While, on average, the eight students as a group uttered twice as many instructions as they did responses, the ratio of Vince's instructions to his responses was more than four to one, CJ's better than seven to one, and Tina's a remarkable 26 to one. The other five students all gave instructions at considerably lower rates, with all but Shirley responding to the directives of others more frequently than they issued such directives themselves.

As Table 3 indicates, students who gave more instructions and fewer responses than the group averages appear also to have achieved greater score gains. The strong correlation ($r = 0.918$, $p = 0.0013$) between student instruction rates and posttest gain scores suggests that the issuing of those instructions may have reflected a particularly productive form of collaboration in a decoding group. To be sure, CJ, Tina and Vince all posted strong pretest scores as well, and the fact that they were both considerably more likely to direct the flow of group activity and less likely to follow others' directions than their peers could certainly reflect higher levels of confidence and group status correlated to their achievement at the start of the unit. Certainly, these students were the most directive and most dominant participants in the majority of the group's decoding efforts; Tina and Vince had by far the largest numbers of total utterances among the eight students, and CJ also spoke far more often than the other two students in Group A. But pretest scores did not uniformly predict this tendency to be directive; while Jessica actually scored a point higher than Vince on the pretest, she gave instructions at a much lower rate, 11% to his 37%, and responses at a considerably higher rate, 18% to his 9%. Trailing only Tina and Vince with her 325 total interactive utterances, she certainly participated extensively in the decoding discourse of her group. But these comparative figures suggest that she took less initiative in guiding the group's activity, and correspondingly managed a considerably smaller posttest gain than the one posted by Vince despite their similar pretest scores.

Comparing Shirley and Monique's posttest and instruction-response data reveals a similar trend. The lowest among the eight students, Shirley's pretest score seems unlikely to predict her as a highly directive figure in her group's decoding efforts. Indeed, she gave instructions much less frequently than Tina, but considerably more

Table 2 Turns coded as instructions and responses

	Total	Tina	CJ	Vince	Jessica	Jason	Reggie	Shirley	Monique
Number of turns	2,416	577	287	425	325	150	63	276	313
Instruction	26%	45%	35%	37%	11%	13%	6%	20%	0%
Response	13%	2%	5%	9%	18%	35%	14%	15%	33%

Table 3 Instructions and gain scores

	Tina	CJ	Vince	Shirley	Jason	Jessica	Monique	Reggie
Instruction	45%	35%	37%	20%	13%	11%	0%	6%
Gain score	1.00	0.88	0.50	0.39	0.38	0.27	0.13	0.08

often than Jessica, at 20 versus 11%. And she gave vastly more instructions than Monique, who issued none. Likewise, her response rate of 15% was just above the average of 13% for all eight students, somewhat below Jessica's 18% and far below Monique's 33%. In other words, the fact that she instructed more and responded less than any of the other students in her group besides Tina seems more likely to align with her high posttest gain than with her low pretest performance.

Jason also gave a higher percentage of instructions, 13 to 6%, than Reggie did. But while these comparative instruction rates might correspond to their respective posttest gains, both students were far below the average rate of 26% for the eight students in the two groups. Moreover, Jason gave responses at a much higher rate, 35%, than the other students who showed large score increases. Thus, while Reggie's rates of instruction and response bear the same apparent relationship with his posttest results as those of the other six students, Jason's relatively low percentage of instructions and his very high percentage of responses appear to make him a notable exception. I suspect, however, that his relative success reflects patterns in his participation that I could not capture by tracking his discursive contributions alone. A soft-spoken boy and hardly the most vocal member of the group, Jason nonetheless played a central and active role in problem solving, nearly always interacting with his PDA even when he seemed not to interact with his group mates. Below, I will examine his contributions to one problem-solving event in greater detail in order to illustrate the nature of his participation and the reasons for his relative success.

Instructions, responses, and publishing: how group roles affect participation

The apparent importance for student learning of taking a directive role in collaborative problem solving speaks to the need for pedagogical structures that allow opportunities for all students to participate in those ways. The assignment of group roles in the *Code Breaker* environment was intended to provide that structure. In particular, the student assigned the role of Publisher assumed a position of potential authority with regard to the group's unfolding strategy. The daily rotation of this role was intended to disrupt hierarchies of status within the group by giving each student regular opportunities to take primary responsibility for coordinating the group's problem-solving process. A student in the Publisher's role might thus be expected to play a more directive role in decoding activities, giving more instructions and responding to fewer than those in other roles.

Contrary to that expectation, the wide variation in students' instruction and response rates displayed in Table 2 suggests that daily rotation of the Publisher role did not lead to equal distributions of such utterances. As Table 4 shows, Publishers across the two focus groups did give instructions at a somewhat higher rate than non-Publishers, 27 to 22%, though they also responded to instructions more frequently, 17 to 10%. In other words, Publishers were more likely to participate

Table 4 Instructions and responses by role

	Total (publishing)	Total (publishing)	Monique (not publishing)	Monique (not publishing)	Tina (publishing)	Tina (not publishing)
Number of turns	669	1,747	55	258	207	370
Instruction	27%	22%	0%	0%	38%	49%
Reponse	17%	10%	60%	27%	2%	1%

in instruction-response dyads than other students, but no more so as givers than as receivers of instructions. Moreover, these variations between Publishers and non-Publishers were modest relative to the variation in rates among individual students, regardless of role. Table 4 highlights the most dramatic case, pairing Tina and Monique, the students with the highest and lowest respective instruction rates. The apparent hierarchy governing those students' respective forms of participation persisted regardless of their time spent in the Publisher role. While 27% of Monique's interactive utterances were responses when she was not in the role of Publisher, already much higher than the average for the eight students, that rate jumped to an astonishing 60% when she was publishing. By striking contrast, a mere 2% of Tina's 207 publishing utterances and 1% of her 370 non-publishing utterances were responses. Likewise, her very high instruction rate of 38% while she was publishing became even higher, 49%, when she was not. In total, Tina issued 260 instructions over three weeks of decoding activities while Monique gave none. Quite simply, Tina gave instructions and Monique responded to them, regardless of their respective role assignments.

Clearly, other factors structuring the interactions among these students influenced their instruction-response dynamic much more forcefully than the pedagogical treatment of role assignments. Indeed, the students accomplished these starkly different discourse patterns by enacting the role of Publisher in quite different ways, as the following excerpts show. The first, in which Tina was the Publisher, finds all of her turns during the segment coded as instructions, while each of her group mates gave only responses (see Table 5).

Table 5 Tina as publisher

Time	Speaker	Turn	Interaction
1:38	Tina	What's the highest...	Instruction
1:40	Tina	Ok who's looking at the ...um,	Instruction
1:43	Monique	Uh, word frequency.	Response
1:44	Tina	Inverse...inverse thing.	Instruction
1:45	Shirley	She is. (points to Jessica)	Response
1:46	Tina	All right. What do you s...	Instruction
1:48	Tina	um, what num...	Instruction
1:50	Tina	No wait, wrong person. Um, word frequency table. All right.	Instruction
1:54	Tina	Do we have a letter that's a positive?	Instruction
1:59	Jessica	Um, x?	Response
2:00	Tina	Do we have a letter, any po..any positive letters?	Instruction
2:01	Monique	We have...no.	Response
2:03	Monique	Letters, but not words.	Response

Throughout this segment, Tina worked to coordinate her group mates and the representations they were assigned to view into an effective problem-solving array. First identifying a piece of information she wanted to acquire—"what's the highest"?—Tina then worked to determine which representation and corresponding classmate could provide that information, and directed that student accordingly. The fact that all of Tina's utterances in this episode are coded as instructions—that they all served to orchestrate the activity of others—might well seem to follow simply from the fact that she was in the Publisher's role. Because it was Tina's job to edit the candidate function, and because her group mates had views of other representations that would inform the ways she would change the function, it stands to reason that she would need to acquire that information from them in order to proceed. Moreover, because responsibility for the candidate function gives the Publisher greater control over the problem-solving process than the other group members, that student certainly might be expected to take leadership in coordinating group activity. As the next segment shows, however, Monique performed the duties of Publisher quite differently (see Table 6).

The division of labor appears to have remained much the same despite the reconfigured group roles. In spite of Monique's move into the Publisher's role, she and Tina appear to have reproduced the pattern of interaction we saw between them in the previous episode. Just as before, four of Monique's comments were responses to instructions given by Tina, and the fifth was a reply to an explanation given by Tina. Likewise, three of Tina's six turns were instructions, while two more were explanations that served to guide Monique as she carried out those instructions.

Tina and Monique thus continued to follow similar patterns of interaction, despite alternating roles, by performing the Publisher duties is very different ways. When Tina served as Publisher, shown in Table 5, she used the distributed vantage points of her group mates to inform her own decisions about how to edit the candidate function. On the other hand, when Monique took over as Publisher, shown in Table 6, it was still Tina, with some help from Shirley, who made the decisions about how the candidate function should be edited, spelling out precisely

Table 6 Monique as publisher

Time	Speaker	Turn	Interaction
1:09	Tina	Change the coefficient to negative one.	Instruction
1:12	Monique	Yeah. Which one's the coefficient?	Response
1:13	Tina	The coefficient is the one before the x.	Explanation
1:16	Monique	Before the x?	Response
1:16	Jessica	The highest number's negative one.	Observation (math)
1:19	Monique	There we go.	Response
1:21	Tina	All right. Now negative two.	Instruction
1:23	Monique	Coefficient.	Response
1:28	Jessica	And the lowest number's negative forty-nine.	Observation (math)
1:40	Tina	Now change the...ummm.	Instruction
1:43	Tina	A is obviously negative one, right?	Explanation
1:45	Monique	Um-hm.	Reply
1:46	Tina	Right now, A is one.	Observation (math)
1:49	Shirley	So you can change that one.	Instruction

what changes Monique should make to which components. In effect, Tina shifted decision-making authority away from the Publisher role and maintained control over the distributed group process from her new assigned screen view. Or, to put it differently, the emergent division of labor into which the students organized themselves appears to have shaped their collaborative work much more powerfully than the one outlined by their teacher and the researchers through the assignment of group roles.

Forms of networked participation

One form of discursive participation in code breaking activities, the giving of instructions, was both linked to gains in posttest achievement, and unaffected by a role assignment intended to support equitable participation in network-supported collaborative problem solving. In this section I document the emergence of other, unscripted forms of participation, facilitated by the network, which may have more effectively supported opportunities for lower-status and lower-achieving students to collaborate and to learn. The excerpts below feature the four focus students who scored lowest on the pre- and posttests, and explore the ways in which the blend of networked personal devices and role-based pedagogy provided those students with access to forms of participation that might, in turn, have supported their learning of mathematics. I will highlight two students, Shirley and Jason, whose relative status in the discourse patterns of their groups belies their strong posttest achievement gains, and contrast their participation with that of another student, Reggie, who was less successful with the posttest. A portrait of a fourth student, Monique, depicts an episode in which her participation in the *Code Breaker* device network provided an opportunity for mathematical learning.

Non-discursive participation

One of the most distinctive aspects of Shirley's and Jason's participation in decoding activities was that these two students appeared to spend a considerable amount of time using their handhelds to engage non-discursively in group problem-solving processes. The following excerpt from a decoding episode with Group B illustrates how Shirley used her device to participate in ways not indicated by her utterances alone. As the segment opened, Monique, the Publisher, was receiving directions from Tina, the Presenter, about how to edit the candidate function, much as we saw in Table 6 above. Meanwhile Shirley, the Equipment Manager, sat hunched forward with her face inches from her PDA screen (Fig. 3). The fingers of her left hand were flattened across the top of the device while her right hand gripped her stylus, which she used to scroll between the graph and her assigned view of the function table:

Tina: Now change the x to squared.
Monique: The who?
Tina: X to squared.
Jessica: Exponent to three.
Tina: Mind you I assume the exponent...which sucks. Ok that's way too big.
Shirley: (staring at her screen) Whoa.
Tina: No, no, no, no. Back, back, back, back back...

Fig 3 Monique, Shirley and Tina

Monique: Back down?

Shirley: (still staring at her screen, addressing no one in particular) What the heck is that?

Tina: Back, back, back, back, back to one, back to one, back to one. See this obviously, it's obviously not times negative one. So try negative two.

[Shirley taps her screen lightly as Tina makes these comments, apparently attempting to highlight values in the graph, and then leans over to glance momentarily at Jessica's screen. After briefly looking back to her own device again, Shirley then props her head up to shift her gaze back and forth between Monique, who edits the candidate function, and Tina, to whom Monique looks up as if waiting for approval]

Tina: The coefficient.

Monique: Oh. (laughs) To a two?

[Shirley fiddles with a pencil for a moment and continues to watch Tina and Monique, and then turns her attention back to her device as Monique begins editing the candidate function again.]

Tina: To a negative two. Negative two!

Monique: I'm going! There. Negative two.

Tina: There we go. Yeah...Now let's change...make the offset up a little.

[Monique adjusts the offset, then looks to Tina for further instructions. Shirley continues to examine her PDA screen, promptly tapping the screen to re-illuminate it when it dimmed from inactivity.]

Tina: Keep going. (motions with her hand for Monique to continue) Wait, wait, wait, stop, stop, stop. I just found out something. Now um,...Um, we shouldn't ever change the offset unless all the letters are solved for. Unless there's no question marks. Because we're only going to be changing, switching the letters around. We're not going to be switching any numbers, we're switching letters. So if they're all question marks, that means we don't change the offset yet.

[Shirley stared intently at Tina throughout this extended explanation, then blurted out the next utterance as soon as Tina paused.]

Shirley: So change it back.

Tina: Change it back.

Shirley: (looks down at her screen for a moment) So, change...

Tina: Now change it back.

Shirley: (looks back up at Tina and speaks firmly) Change the constant.

Tina: Yeah, change the...Good idea! I forgot all about that!

Tina is clearly the most active discursive participant in this episode; she takes 12 speaking turns, some of which are quite lengthy. Shirley actually has five comments to Monique's four, but whereas Monique dialogued with Tina throughout the segment, Shirley had only spoken twice prior to the final moments of the excerpt. Moreover, she appeared to address those first two remarks only to herself, as she looked to her device rather than her classmates as she spoke, and none of the other girls acknowledged those utterances. Likewise, the first 11 of Tina's 12 turns, as well as the single contribution from Jessica, were all directed at Monique, whereas only Tina's final comment addressed Shirley. In other words, Shirley appeared to be positioned firmly on the discursive periphery of a problem-solving process directed by Tina and implemented by Monique.

Nonetheless, Shirley closely monitored that process. Her eyes were nearly always focused on either her own handheld screen, or on Tina and Monique as they discussed changes to the candidate function. Though she was not editing that function herself, she actively engaged the device, even departing from her own assigned station to better track their progress. Ultimately, this careful attention to the unfolding details of the problem appears to have positioned her to make a decisive contribution when she did finally enter into the conversation between the other girls. Tina's extended comment about the offset[3] proved to be a pivotal insight not only into breaking this code, but also into the group's decryption strategy more generally. That context makes Shirley's direction to "change [the offset] back [to zero]" particularly noteworthy, because that comment indicates that Shirley had not only followed the fairly intricate logic articulated by Tina, but also beaten Tina to the punch in translating that logic into a course of action.

Shirley's subsequent assertion that they should "change the constant" provides another indication that she had been quietly but actively investigating a solution path of her own. Tina had directed Monique to edit first the exponent, then the coefficient, then the offset to specific values, but failed to realize that the candidate function also had an incorrect constant value. The fact that Shirley spotted the error suggests that she had indeed kept up with the changes made by Tina and Monique throughout the segment—that she had been a quiet but active participant. Having agreed that they needed to adjust the constant, the girls quickly determined the correct value, and successfully decoded the message soon after.

This episode provides some insight into the ways Shirley participated, with and without utterances, in the group's problem-solving process. That non-discursive participation appears to have been uniquely facilitated by the handheld network; Shirley was not only able to listen to the conversation between Tina and Monique, but also to follow the consequences of their decisions, and to consider alternatives, for a set of shared objects on her own device. While a group task in a paper-and-pencil environment would certainly have allowed her to explore her own ideas while other members of the group pursued a joint strategy, it would have necessitated decoupling the relevant artifacts—equations, sketches, calculations—she manipulated from those shared by the other girls. A personal device linked to a group network allowed Shirley to work independently of the other girls' joint enterprise, but to do so with objects that changed in common with that enterprise. By attending to those shared objects and to the other girls' discourse even as she maintained her own investigation, she was ultimately able both to join the discussion by building on Tina's observation regarding offsets, and then to redirect that discussion toward her own line of inquiry.

Coordinating discourse around shared objects

Just as the previous example found Shirley using her PDA to engage much more actively in her group's decoding efforts than her utterances alone might indicate, the next excerpt (Table 7) suggests that Jason came to play a more directive role than the coding scheme reveals. While Shirley found ways to participate non-discursively by engaging shared objects through her device, here Jason worked together with Vince to coordinate their discourse around shared objects. Vince occupied the Publisher's role in this episode, so his device displayed the encoded text and the equation for the candidate function. As the Equipment Manager, Jason viewed the frequency and function tables. The segment opens with Vince asking Jason whether any of the candidate values in the function table were "close" to those of the actual encrypted message, as shown in the frequency table.

Vince was clearly dissatisfied with Jason's response (line 2:37) to his question about being "close". Jason's report that there was "a frequency for...nothing" indicated that his interpretation of the frequency table differed from Vince's intended use of the representation. In grabbing Jason's PDA and looking at the

Table 7 Aligning discourse t7.1

2:35	Vince	Am I close?	Instruction	t7.2
2:37	Jason	You have a frequency for...nothing.	Response	t7.3
2:39	Vince	What the...can I see this? (Reaches for Jason's PDA). OK, I...duh. I am way off. OK, what's the highest number here?	Instruction	t7.4
2:47a	Jason	One...	Response	t7.5
2:47b	Vince	I'm too high.		t7.6
2:48	Jason	Huge.	Response	t7.7
2:49	Vince	Too high.		t7.8
2:56	Jason	All right, you're pretty close.		t7.9
2:57	Vince	I am?		t7.10
2:58	Jason	Go back.	Instruction	t7.11
3:00	Vince	Here's what I want you to tell me.	Instruction	t7.12
3:01	Vince	OK. (Pointing to Jason's PDA). Right here, OK, 39556, right? Look for a number close around.	Instruction	t7.13
3:07	Jason	Like, right here, before when it was three hundred thirty-three thousand something it was pretty close to this.	Response	t7.14
3:13	Vince	OK. Tell me if I'm close.	Instruction	t7.15
3:16	Vince	How's that?	Instruction	t7.16
3:17	Jason	Uhhh, mine hasn't loaded yet.	Response	t7.17
3:18	Reggie	Pretty close.	Response	t7.18
3:19	Jason	Yeah, it's close.	Response	t7.19
3:21	Vince	Do I have any uhh...	Instruction	t7.20
3:22	Jason	Yeah, you have a match, right here.	Response	t7.21
3:23	Vince	I do? One?	Instruction	t7.22
3:26	Jason	Yeah.	Response	t7.23
3:26	Vince	Ok. How about now?	Instruction	t7.24
3:27	Jason	Wait.	Response	t7.25
3:30	Jason	Um, it's a little bit off...	Response	t7.26
3:38	Jason	Keep on going...	Instruction	t7.27
3:39	Jason	Off, down.	Instruction	t7.28

tables himself, Vince initially appeared to be attempting to bypass Jason altogether, determining on his own that his candidate function was "way off". Jason, however, kept a hand on his device even as Vince pulled it to the meeting point between their two desks. Consequently, the two boys examined the same screen together as Vince sought to compare the highest numbers in the frequency and function tables. Each boy had one hand on the PDA as Vince reached in with his other hand to point a finger at the frequency table and ask Jason "what's the highest number here"? Vince could, and did, read this number himself; that he nonetheless instructed Jason to do the same likely indicates an effort to align their respective interpretations of the tables. Moving his finger up Jason's PDA screen to the function table, Vince pointed again as he announced, "I'm too high", and turned back to his own PDA, repeating that his current candidate had made the values in the function table "too high".

This temporary collapsing of the distributed network down to a single device appears to have allowed Vince and Jason to coordinate their use of the shared mathematical objects represented in these tables. As Vince turned back to his own device and adjusted the candidate function according to this latest analysis, Jason reported in line 2:56 that they were now "pretty close". This unsolicited observation marks a dramatic shift from the feedback Jason had given a few moments before. Seconds earlier, when Vince had called in line 2:35 for a comparison between values in two tables, Jason responded by attempting to share information from one of those representational artifacts without reference to the other. But after the boys' brief consultation around a shared device, Jason was able to provide just the kind of comparative observation Vince had initially asked for.

On hearing Jason's report that he was "pretty close" and his subsequent instruction to "go back", Vince appeared surprised. He mused, "I am"? in line 2:57, and then moved to give further guidance in line 3:00: "here's what I want you to tell me". Again leaning across his desk and putting his right hand on Jason's PDA, Vince tilted the device toward himself, and stretched the index finger of his left hand toward the right column of the frequency table as he made the reference in line 3:01: "right here, OK, three-nine-five-five-six, right"? He then slid the finger up the screen, apparently sweeping it across the function table as he directed Jason to "look for a number close around". In response, Jason pointed to the function table with his own left index finger (Fig. 4) as he indicated "like, right here" in line 3:07, and then curled the finger down to indicate the frequency table as he noted that the number had been "pretty close to this". Apparently satisfied with their calibration, Vince sat back in his seat and turned his attention back to his own computer as he asked Jason to tell

Figure 4 Reggie, Vince and Jason

him “if I’m close” (line 3:13). Now, it appeared, they had established an agreed-upon meaning for “close”—as a descriptor of the relationship between a pair of values, one in each of the function and frequency tables. Thus equipped, they were able to proceed through the rest of the segment looking at their respective devices, each showing a different view, with a common discursive framework for directing their subsequent efforts to interpret the tables and revise the candidate function.

This exchange highlights the precision with which the boys worked to align their discourse in order to determine how to edit the candidate function appropriately. Doing so required the boys twice coming together to examine a shared device in order to establish agreement about shared objects they would then coordinate on their separate devices. That discursive alignment also appears to have enabled a shift in the way Jason was able to participate in the boys’ joint problem-solving process. Of Vince’s 15 utterances in this excerpt, nine were coded as instructions, and none as responses. By contrast, only three of Jason’s 14 utterances were coded as instructions, and ten as responses. The quality of Jason’s utterances, however, underwent an important shift over the course of the brief episode. His first three responses, in lines 2:37, 2:47a, and 2:48, amounted to routine efforts at reading values from the tables in his assigned view. In lines 2:56 and 2:58, he attempted to interpret what he saw, and to advise Vince accordingly, but the boys had not yet fully established a shared meaning for those inferences. By line 3:19, however, he was able to assert that the candidate was “close” without being questioned by Vince, to identify a “match” between a value in the frequency table and one in the function table (line 3:22), and to give fairly specific directions about the next adjustments Vince should make to the coefficient of the candidate function (lines 3:38 and 3:39). In other words, Jason had progressed from simply extending Vince’s view to the representations on both their screens, to actively drawing his own inferences and recommending courses of action.

Reggie’s sole turn in this excerpt, in line 3:18, provides an informative contrast to Jason’s shifting participation. When Jason was not able to reply immediately due to an updating delay on his computer (line 3:17), Reggie chimed in with his own report that they were “pretty close”. Careful examination of the video record, however, indicates that Reggie’s use of “close” was not aligned with the joint meaning regarding shared objects that Vince and Jason had established in the preceding turns. Reggie appeared mostly distracted during Jason and Vince’s discussion, talking to another student and then staring at something off-camera. When Vince prompted, “how’s that”? in line 3:16, Reggie looked down at his screen just as it darkened into its low-power mode from disuse. It was at this dimmed screen that he glanced as he called out “pretty close” before tapping the screen to re-illuminate it. Though he may have managed to catch a glimpse of something on which to base this report, it appears more likely that he was simply “playing along”, uttering a phrase that fit the pattern of the group’s discourse so that he could maintain the appearance of participation. He had not, after all, participated in the work done by Vince and Jason to reach agreement on the meaning of “close” or the objects it referenced. In effect, he was attempting to participate discursively in the work of the group without engaging the shared objects around which that discourse was organized.

In a way, then, the handheld network created an opportunity for Reggie to feign engagement. In looking at his device and repeating a phrase Jason had used earlier, he appears to have been attempting to mask the fact that he was not keeping up with the problem-solving steps taken by the other boys. Jointly looking at one device with a group mate provided a way for Jason to jointly engage, and to make better sense of, the shared mathematical objects of the handheld network. Conversely, looking at a device without jointly engaging those shared objects allowed Reggie to hide the fact that he wasn't really participating in the collaborative endeavor, and likely also that he didn't share in the collective understanding of those objects' meaning.

Highly discursive participation

The previous examples highlight ways students used the handheld network to circumvent or mediate discursive interactions by engaging in transactions across their linked devices. By contrast, the following episode draws attention to ways that the blend of networked personal devices and role-based pedagogy augmented discursive interactions among group members. Students' assigned views often required them to participate in ways I characterize as highly discursive. Such circumstances might require a student to speak in ways requested and closely scrutinized by group mates who relied on her view, and to use her utterances as a bridge between other students and objects on her own device. The episode that follows finds Monique speaking under such pressures. Unlike Shirley and Jason, Monique achieved neither high-status forms of discursive participation nor a large posttest gain over the course of the *Code Breaker* unit. Nonetheless, in the excerpts below she appears to have interacted with her group mates to accomplish some meaningful mathematical learning. The first of those excerpts finds Group B in the midst of an effort to decode a message that contained predominantly negative values (see Fig. 2 for *Code Breaker* views of this message, encrypted with the function $y = -4x + 7$):

Tina: What's the highest...number in the word frequency table? I mean, what's the lowest?
Monique: Umm...three.
Tina: That's the highest. Cause they're all negative.
Monique: Oh yeah, so positive. Ok.
Jessica: It's forty-one.
Tina: I'm looking at negative seventy-seven.
Monique: The lowest... Negative twenty-five.
Jessica: Oh. No, I got...well, never mind.
Monique: No, negative one.

Tina, the Publisher, began the excerpt by asking Monique, the Recorder, to report the lowest number in the word frequency table. Because all the numbers but one in this code were negative, this task caused some confusion for Monique, who began by offering an answer of three. Jessica, the Equipment Manager, spotted a −41 in the frequency table and offered it up as an alternative candidate. Tina announced a lower value, reading a "negative seventy-seven" from the coded text. Even as Tina volunteered this candidate, Monique scanned the word frequency table for other values that might be even lower, next offering up "negative twenty-five". A moment later, she reported that she had found a "negative one," which she thought to be lower still. In each of these instances, she mistook the successively lower absolute values of −77, −25, and −1 for successively lower relative values. The next transcript segment picks up precisely where the last left off, and finds Jessica and Shirley reacting to Monique's report:

Jessica: No the, the...
Shirley: No, no, that's not the...
Jessica: The higher the negative, the, the lower...the positive.
Monique: Oh then it's, then it's negative...seventy-seven.
Jessica: I got...
Shirley: Seventy-seven? (looks at her own PDA as if to confirm)
Monique: Yeah. (Several seconds pass). I just learned something. Cool.

After Monique settled on negative one as the lowest number in the code, Jessica and Shirley moved quickly to correct her. Jessica's explanation, "the higher the negative...the lower the positive", did the trick; on hearing it, Monique turned her attention back to the word frequency table, scanning for a few seconds before correctly identifying "negative... seventy-seven" as the lowest value. Moreover, she appeared sufficiently confident about her interpretation of this number as the lowest value that she affirmed her report—after studying the word frequency table for a few moments more—when Shirley questioned it. As the rest of the group continued analyzing the code, Monique leaned back in her chair, and addressed no one in particular as she announced that she had "just learned something".

This self-assessment appears correct; faced with a similar situation while breaking a new code a week later, Monique had no difficulty picking out the number of lowest absolute value as the one of greatest relative value in the set. Inasmuch as she had learned something in this episode, I would argue that the opportunity for that learning emerged directly from the collaborative pedagogical context and the circumstance of coordinating dynamically linked representations across networked devices. Because her group role called on her to share certain kinds of information provided by her unique view of the software, Monique was put into the position of having to articulate an interpretation of that view. The collective work of the group depended on her report, and when her group mates suspected confusion on her part, they were sufficiently invested in either correcting the report, or helping her learn, or both, that they intervened. While this may simply reflect good group work, I believe that effectively collaborating in this handheld code-breaking system placed unique discursive demands on the students. As they tried to coordinate representations that were linked across devices but not shared in each student's assigned view, students were often pressured to voice shaky ideas, as Monique did in trying to identify the lowest number, or to evaluate and provide feedback on those ideas voiced by others, as Shirley and Jessica did in response. This need to clearly and accurately communicate about the shared mathematical objects on their devices may foster important learning opportunities for students, and not only those students who might tend to strategize and direct the problem-solving activity of a group.

Conclusion

The episodes above indicate several ways that a classroom device network can expand the range of collaborative "frequencies" through which students participate in a small group. In the absence of networked devices, members of a group presumably interact and coordinate problem-solving efforts primarily discursively, through shared utterances.[4] As the analysis of discourse among students in these focus groups indicates, those shared utterances may reflect hierarchies in status that structure the kinds of contributions made by different members of a group. Those hierarchies in discursive participation were closely linked not only to levels of student achievement, but also to gains in that achievement over the course of the study. Moreover, an assigned student role intended to equalize student contributions over time by rotating authority among members in fact left those status hierarchies among students' forms of speech relatively undisturbed.

But these students did more than speak; they simultaneously negotiated shared utterances through a discursive network and shared objects through a device network. The cases of Shirley, Jason and Monique suggest that the classroom network may have broadened the relatively narrow bandwidth of participatory forms provided through discourse to create additional ways for students to engage in, contribute to, and learn from a collaborative activity. While Shirley used the shared objects depicted on her device to engage non-discursively in the work of her group, Jason facilitated his discursive participation by coordinating his interpretation of those shared objects with Vince, and the discursive pressures of reporting on the particular aspects of a collective artifact associated with her group role facilitated Monique's moment of self-conscious mathematical learning. Taken together, these cases suggest ways that a network might add to the participatory opportunities of classroom collaboration.

Importantly, the collective artifacts mediating students' discursive interactions in each case were not only network-based, but also mathematically rich. Interactions through the network were not simply additional chances to speak, but rather opportunities to engage in or even alter the course of an open-ended problem-solving task. As such, the students' common focus on those shared artifacts may have engendered forms of participation particularly supportive of mathematical learning.

Though the device network appears to have played a part in facilitating the participation, and perhaps enhancing the achievement, of some lower-status and lower-performing students in these focus groups, the fundamental gap between higher and lower-attaining students remained unchanged. Variations in gains notwithstanding, the four highest-scoring students on the pretest likewise managed the four highest scores on the posttest. Three of those four were also the most likely to give instructions and the least likely to respond to them, regardless of the intended status treatment associated with the Publisher role. And Reggie, the student with the smallest posttest gain among the eight, actually used his networked device to minimize rather than facilitate his own participation.

The case of Reggie, in particular, suggests that the particular distribution of roles and representations used here may not have adequately scaffolded participatory opportunities for all members of a group. Likewise, Shirley's strategies for non-discursive participation included examining representations other than those she had been assigned. The lessons of these and other episodes may yield subsequent designs for networked collaboration that go considerably further in broadening opportunities to engage in complex problem-solving activity, though challenges will undoubtedly remain. Hierarchies in students' social and academic status can be profoundly persistent, and the best blends of networked technology and group pedagogy may still be only modest interventions. But the difficulty of bridging gaps in achievement only emphasizes the need for designs that provide all students with an array of resources for disrupting those hierarchies. The forms of networked participation outlined here offer one set of steps toward widening that array.

Acknowledgments Support for this project was provided by Stanford University, The Wallenberg Global Learning Network, and Hewlett-Packard Corporation. Thanks to Karin Brodie and to Melissa Eiler White for insightful comments on an earlier draft.

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